Memory and Tail effects with interacting scalars

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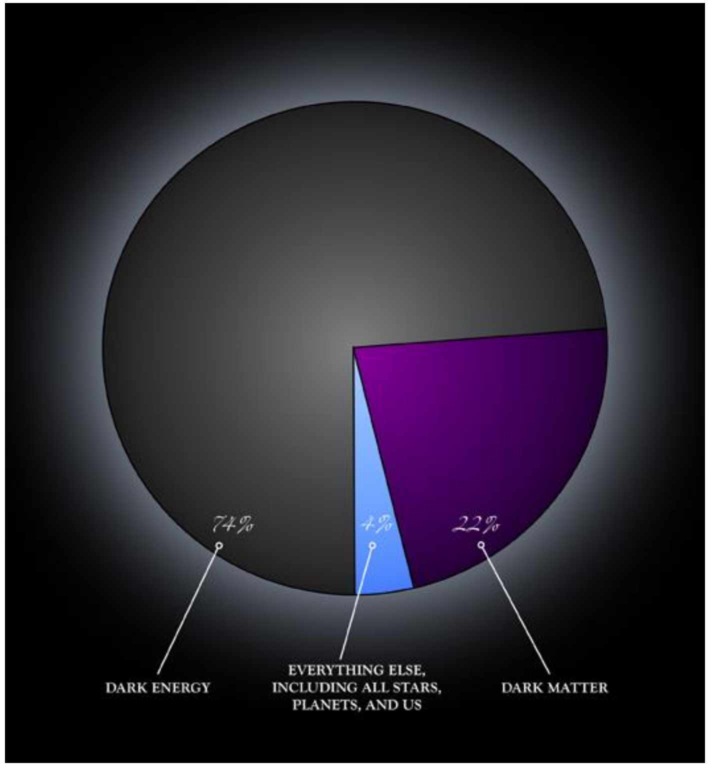
No good understanding of dominant phenomena on cosmological scales where both dark matter and dark energy are necessary to explain Baryon Acoustic Oscillations (BAO) or the Cosmic Microwave Background (CMB).

DARK MATTER is the dominant form of matter in the Universe but no one knows what it could be.



$$S_{\Lambda\mathrm{CDM}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}(R-2\Lambda)$$

Cosmological constant

The cosmological constant can be replaced by dynamical fields with more fundamental origins:

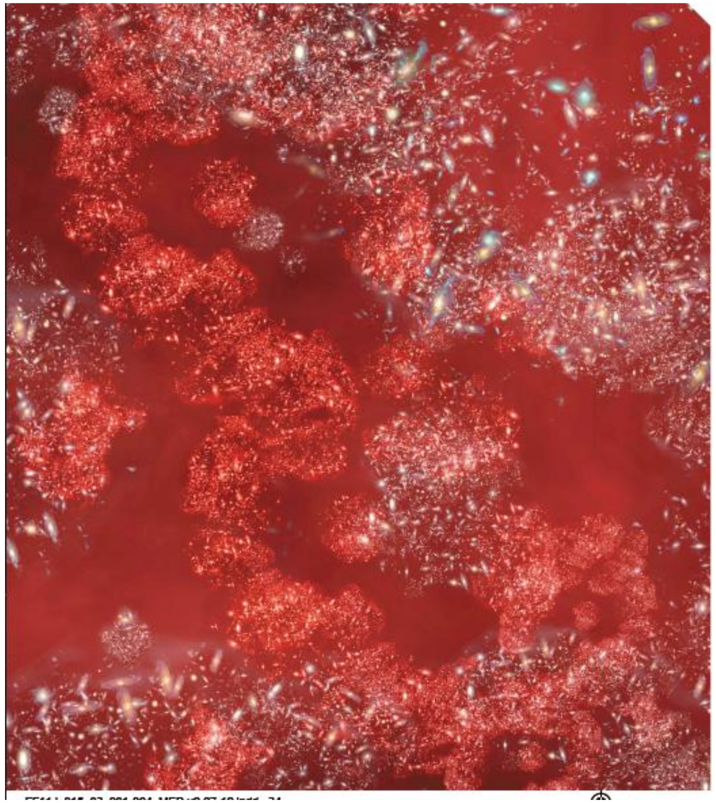
DARK ENERGY

In cosmology in the last twenty five years, we have built the standard model based on 95 % of unknown entities. This is particularly severe for DARK ENERGY:

$$\Lambda \to \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$$

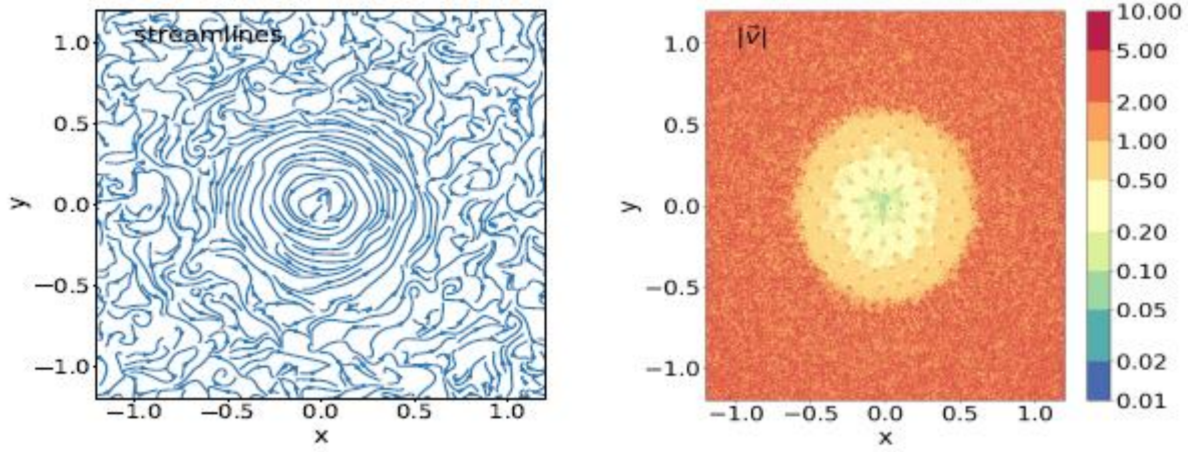
going from a cosmological constant to a dark energy field (DESI?).

This field is generically coupled to matter (otherwise fine-tuning) and nearly massless (to generate the acceleration of the expansion)



Dark matter could also be extremely light and made out of scalar field.

$$V(\phi) \supset \frac{\lambda_4}{4}\phi^4$$



Self interactions lead to solitons, vortices. All relevant for galactic physics.

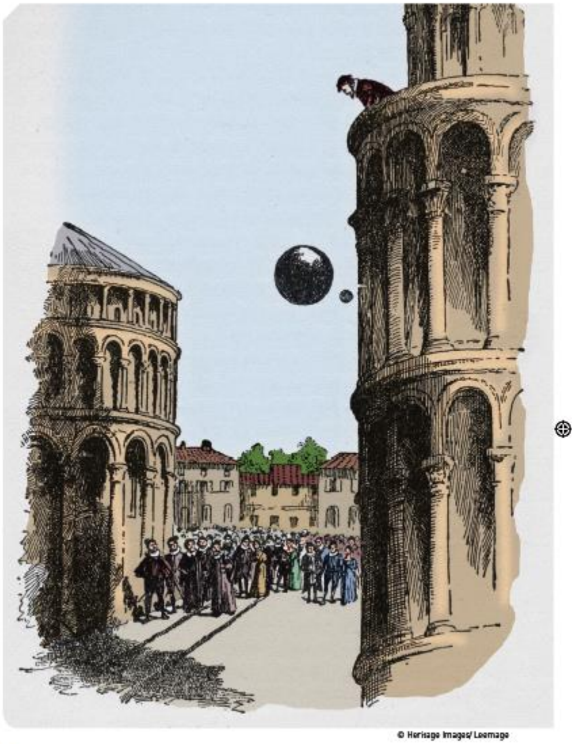
The interactions with matter are captured by a single metric.

$$S_m(\psi_i, \tilde{g}_{\mu\nu})$$

Matter interacts with the Jordan metric which is not the dynamical one in the Einstein equation. The Bekenstein form of the Jordan metric is:

$$\tilde{g}_{\mu\nu} = A^2(\phi, X) g_{\mu\nu} + B^2(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

It preserves the diffeomorphism invariance and does not lead to higher derivative Lagrangian (ghosts).



If the scalar field is ver light, one of its main effects will depend on its interactions with matter:

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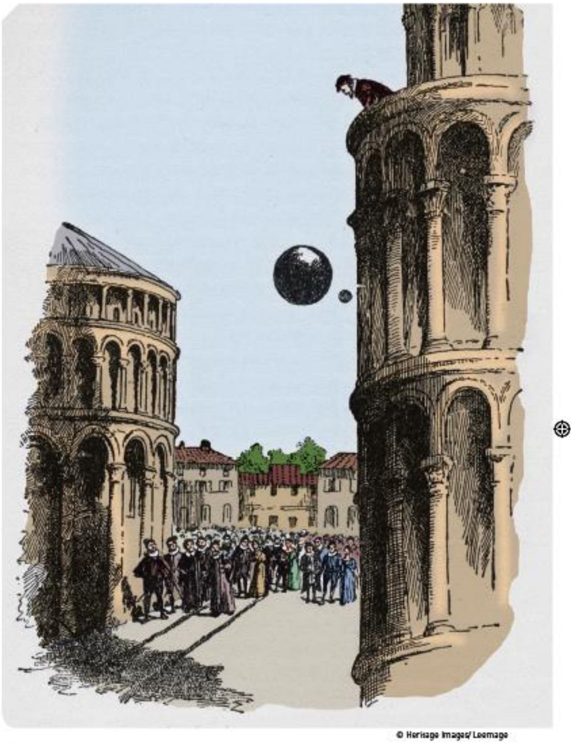
Conformal coupling

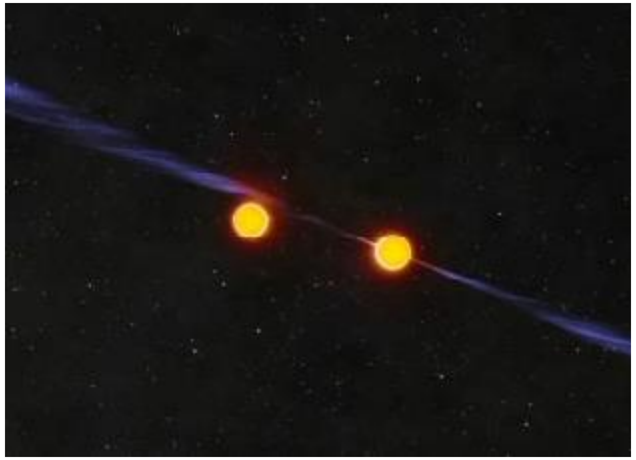
Disformal coupling



$$X = -\frac{1}{2}(\partial\phi)^2$$

Conformal case already hallowed ground , focus mainly on disformal below.

To simplify the discussion, we will focus on a Yukawa interaction and a constant disformal term:



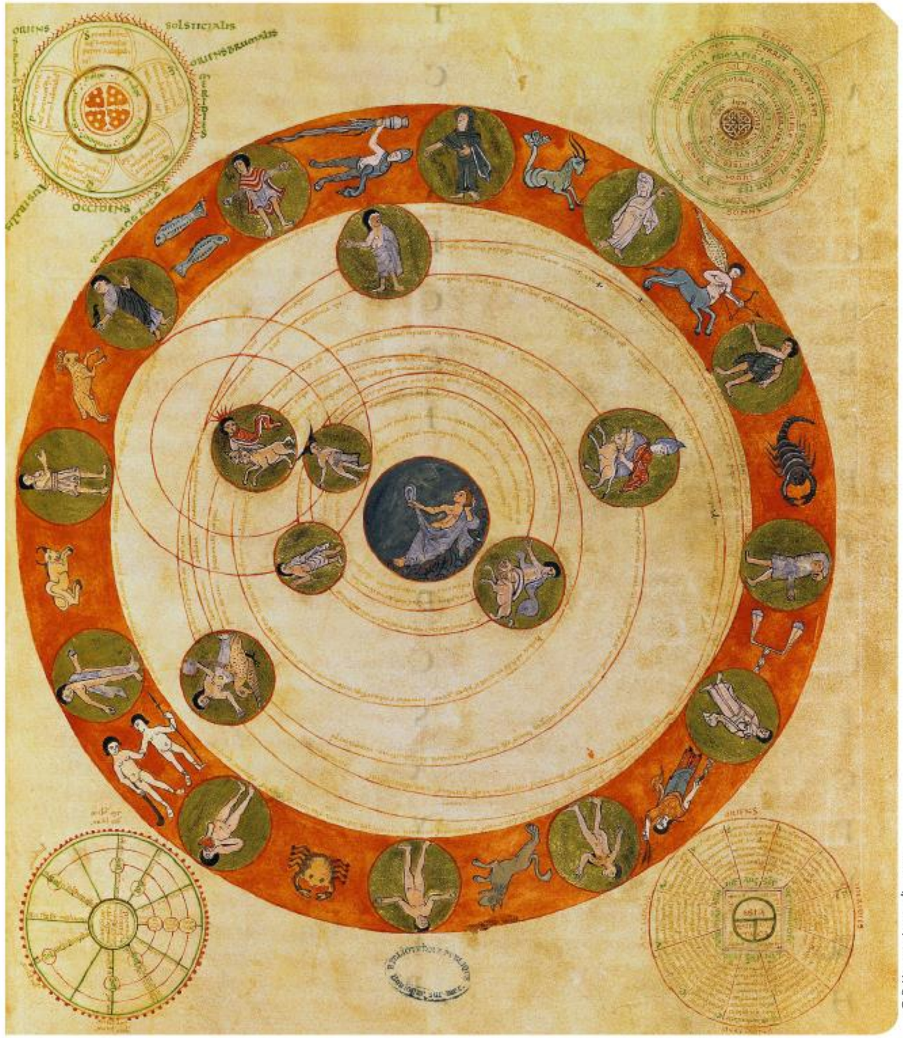
$$A(\phi) = e^{\beta\phi/m_{\rm Pl}},\ B(\phi) = \frac{\sqrt{2}}{m_{\rm Pl}\Lambda}$$

This is motivated by the case of massive gravity where:

$$\beta = \frac{1}{\sqrt{6}},\ \Lambda \simeq m_{\rm graviton}$$

$\beta^2 \leq 10^{-5}$ Cassini bound

We will test interactions with matter and self-interactions using gravitational effects



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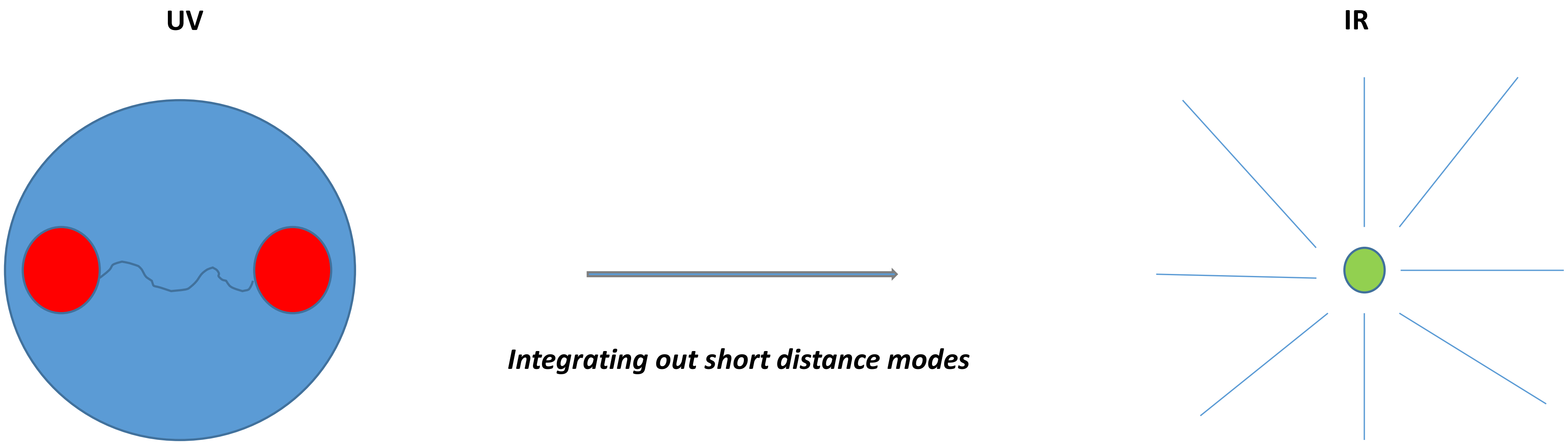
For the interaction with matter, we will consider the case of hyperbolic orbits and the possible scalar memory effects.

For the self-interactions, tail effects where non-local time correlation occur are particularly interesting.

The effective approach



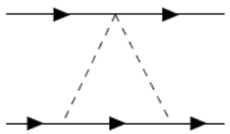
A Two body system consisting of two nearly point-like particles is viewed from afar as a blob with many non-vanishing multipoles.

The effective action for a two body system can be obtained symbolically (in the in-out picture) as:

$$e^{iS_{\text{eff}}[x_\alpha]} = \int \mathcal{D}h_{\mu\nu}\mathcal{D}\phi e^{i(S_{EH}+S_\phi+S_m)}$$

where the metric is expanded around Minkowski space and the gravitons and scalars are integrated out. The effective action depends only on the positions and velocities of the bodies. This gives the ***conservative*** dynamics of the system.

The effective action can be obtained by two methods:



A ***diagrammatic*** approach where vertices and propagators involve the interaction between on scalars and gravitons with the external bodies in a non-relativistic approximation. No external graviton or scalar legs are taken into account.

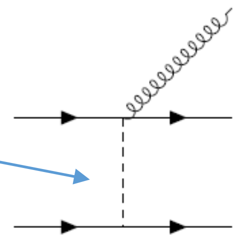
An ***algebraic*** approach where the equations of motion are solved and the solutions used to calculate the action (Fokker)

The system is dissipative as energy is radiated away by gravitons and scalars: $h_{\mu\nu} = \bar{h}_{\mu\nu} + H_{\mu\nu}, \quad \phi = \bar{\phi} + \Phi$

External fields radiated away

$$e^{iS_{\text{eff}}[x_\alpha, \bar{h}_{\mu\nu}, \bar{\phi}]} = \int \mathcal{D}H_{\mu\nu}\mathcal{D}\Phi e^{i(S_{EH}+S_\phi+S_m)}$$

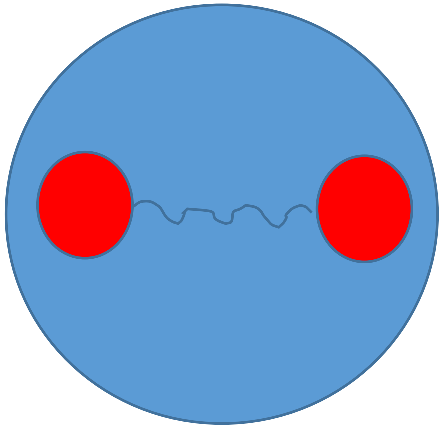
Integrating out short distances



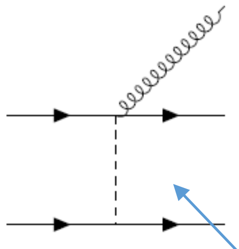
Integrating out the radiated field leads to an imaginary part in the effective action: ***dissipation***

$$e^{iS_{\text{eff}}[x_\alpha]} = \int \mathcal{D}\bar{h}_{\mu\nu}\mathcal{D}\bar{\phi} e^{iS_{\text{eff}}[x_\alpha, \bar{h}_{\mu\nu}, \bar{\phi}]}$$

Contains both conservative and dissipative parts.



The long-distance action:



$$P_\phi = 2G_N[< \dot{I}^2 > + \frac{1}{3} < \ddot{I}^i \ddot{I}^i > + \frac{1}{30} < \dddot{I}^{ij} \dddot{I}^{ij} >]$$

After integration over the internal NR scalars the ***effective action*** for the radiated scalar is:

$$S_{\rm eff} = \int d^4x \left(-\frac{1}{2}(\partial\bar{\phi})^2 + \frac{J}{m_{\rm Pl}}\bar{\phi}\right)$$

The effective interactions between the particles and the scalars are replaced by a source tem J

Expanding around the centre of mass this gives the effective action at quadratic order:

$$S_{\rm eff} \supset \int dt \left(I\bar{\phi} + I^i \partial_i \bar{\phi} + \frac{1}{2} I^{ij} \partial_i \partial_j \bar{\phi}\right)$$

$$I = \int d^3x \left(J + \frac{1}{6}\partial_t^2 J x^2\right), \;\; I^i = \int d^3x J x^i, \;\; I^{ij} = \int d^3x \left(x^i x^j - \frac{1}{3} x^2 \delta^{ij}\right)$$

Integrating out the scalar and taking the imaginary part gives the emitted power.

A tale of three propagators:

For internal lines the scalars are off-shell in the *non-relativistic* approximation:

$$\frac{1}{k_\mu k^\mu - i\epsilon} = \frac{1}{\vec{k}^2}(1 + \frac{k_0^2}{\vec{k}^2} + \ldots)$$

In the effective action calculation, the Feynman propagator leads to an *imaginary* part:

$$\frac{1}{k_\mu k^\mu - i\epsilon} = P(\frac{1}{k_\mu k^\mu}) + i\pi\delta(k^\mu k_\mu)$$

For the Fokker method and the in-in calculations, the *retarded* propagator:

$$\frac{1}{\vec{k}^2 - (k_0 - i\epsilon)^2} \to -\frac{1}{2\pi}\theta(x_0 - x_0')\delta((x - x')^2)$$

This is the multipole expansion of the emitted power in monopole, dipole and quadrupole terms. The conformal and disformal expressions for the source J can be worked out using the conformal and disformal vertices. There are several contributions:

$$J_{v^0} = -\beta(m_A\delta(\vec{x}-\vec{x}_A) + m_B\delta(\vec{x}-\vec{x}_B))$$

Direct conformal coupling

$$J_{v^2} = \beta(m_A\frac{\vec{v}_A^2}{2}\delta(\vec{x}-\vec{x}_A)+m_B\frac{\vec{v}_B^2}{2}\delta(\vec{x}-\vec{x}_B))+\frac{\beta G_N m_A m_B}{|\vec{x}_A-\vec{x}_B|}(\delta(\vec{x}-\vec{x}_A)+\delta(\vec{x}-\vec{x}_B))$$

Expansion of proper time of one particle

One graviton exchange

The disformal interaction:

$$J_{\rm dis} = 4\beta \frac{G_N m_A m_B}{\Lambda^2} \frac{d^2}{dt^2} \left(\frac{1}{|\vec{x}_A - \vec{x}_B|}\right)(\delta(\vec{x} - \vec{x}_A) + \delta(\vec{x} - \vec{x}_B))$$

Only contributes to the monopole at this order. Quadratic in the velocities.

This allows one to calculate the monopole, dipole and quadrupole at lowest order in the velocities:

$$I^{ij} = -\beta(m_A(x^i x^j - \frac{1}{3}x^2\delta^{ij} + (A \to B))$$

Leading to the quadrupole emission (small).

The dipole is not modified by the disformal coupling and corresponds to the usual scalar-tensor result:

$$I^i = -\beta(m_A x_A^i + m_B x_B^i) - 4\beta \frac{G_N \mu m}{\Lambda^2} \Delta r^i \frac{d^2}{dt^2}(\frac{1}{r})$$

Vanishes by the centre of mass theorem (no motion of centre of mass)

The monopole is where both the conformal and disformal interactions enter:

$$I_{v^0} = -\beta(m_A + m_B)$$

Gives vanishing effects because conservation of matter

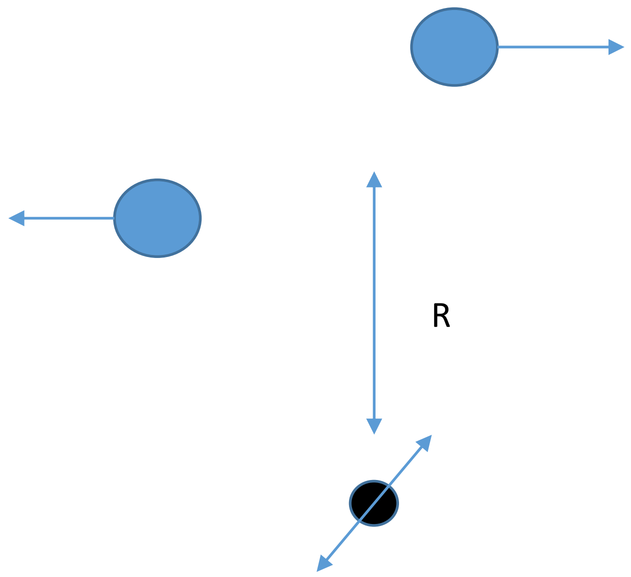
$$I_{v^2} = 4\beta G_N m_A m_B (\frac{2+\beta^2}{3|\vec{x}_A - \vec{x}_B|} + \frac{2}{\Lambda^2}\frac{d^2}{dt^2}(\frac{1}{|\vec{x}_A - \vec{x}_B|}))$$

Leads to new contributions to the emitted power.

Hyperbolic encounters and memory effects

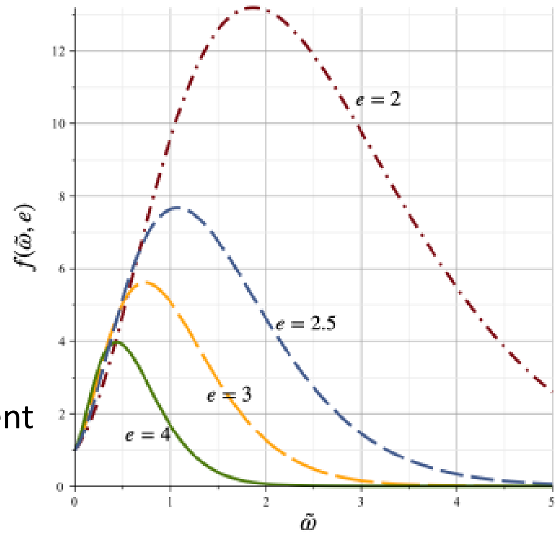
When two objects go past each other, in GR there is a linear memory effect.

Related to the emitted power in gravitational waves at zero frequency



Variation of the rod length

Displacement



Emitted power spectrum

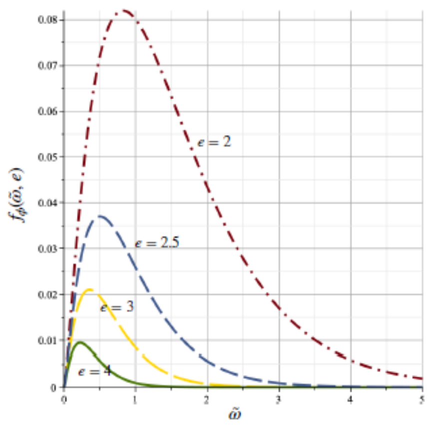
$$\delta^{(2)}_{\rm GR} = -\frac{4G_N\mu}{R} v_\infty^2 \frac{\sqrt{e^2-1}}{e^2}(l_x l_y - (N.l)(l_x N_y + l_y N_x) + \frac{1}{2}((N.l)^2+1)N_x N_y))$$

The scalar memory

The Jacobi deviation equation:

$$\frac{d^2\xi^i}{dt^2} = -R^i_{0j0}\xi^j$$

Riemann tensor of Jordan metric



Monopole emission

Far away the scalar field radiates:

$$\phi = \frac{\mathcal{Q}(t-R)}{4\pi m_{\rm Pl} R}$$

$$\delta_\phi = 2G_N\beta\frac{(1-(N.l)^2)}{R}(\mathcal{Q}(+\infty)-\mathcal{Q}(-\infty))$$

The effective charge is given in terms of the multipoles

$$\mathcal{Q} = I + \dot{I}_i N^i + \frac{1}{2}\ddot{I}_{ij}N^iN^j$$

$$\delta_\phi = -8\beta^2\frac{G_N\mu}{R}(1-(N.l)^2)N_xN_y\frac{\sqrt{e^2-1}}{e^2}v^2_\infty$$

Surprinsingly, the disformal interaction disappears from the memory effect and power at zero frequency…. An explanation is the longitudinal nature of the disformal metric compared to the transverse component of the conformal interaction. In fact the result is related to the BMS formalism.

Where can you see the disformal effects?

Scalar Kicks

$$v_{cm} = \frac{465}{105}\Delta\nu^2\left(\frac{G_N m}{p}\right)^4$$

$$\Delta = \frac{m_A - m_B}{m}, \quad \nu = \frac{m_A m_B}{m^2}$$

$$\frac{dP^i}{dt} = -F^i_{\rm GR} - \frac{4G_N}{3}\dot{I}\ddot{I}^i - \frac{8G_N}{15}\dot{I}_j\ddot{I}^{ji}$$

The centre of mass is displaced due to radiation.

$$\begin{aligned} f^y_\phi \ =\ & \frac{1}{41760e}\left[\sqrt{e^2-1}(1783057e^8 + 10915688e^6 + 9108564e^4 + 942560e^2 - 2144)\right. \\ & \left. +150255e^2\arccos(-1/e)\left(e^8 + \frac{14090}{477}e^6 + \frac{38480}{477}e^4 + \frac{2016}{53}e^2 + \frac{1024}{477}\right)\right] \end{aligned}$$

$$\Delta v^\phi_y = \epsilon_\Lambda v_{\rm cm} f^y_\phi$$

$$\Delta V^y_{\rm GR} = \frac{5943}{1392}v_{cm}\left[\sqrt{1-\frac{1}{e^2}}\left(e^4 + \frac{751}{447}e^2 + \frac{32}{447}\right) + \frac{37}{298}e^2\arccos(-1/e)\left(e^4 + \frac{456}{37}e^2 + \frac{312}{27}\right)\right]$$

The centre of mass kick is proportional to the disformal effect

$$v_y^{\phi} \propto \epsilon_{\Lambda} v_y^{\mathrm{GR}}$$

$$\epsilon_{\Lambda} = \frac{\beta^2 G_N m}{\Lambda^2 p^3}$$

The most favourable case for observation of this scalar kick would be white dwarfs when the closest approach is given by the cut-off scale determined by:

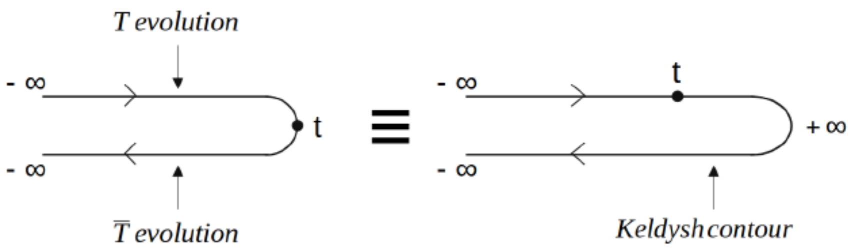
$$\Lambda \gtrsim \frac{10^{-10}}{\sqrt{\beta}} \mathrm{eV}$$

If one were unbelievably optimistic, the observation of scalar memory and kicks would distinguish between conformal and disformal interactions…

The long distance action revisited

The conservative and dissipative effects can be captured easily in the in-in formalism of Schwinger-Keldysh:

$$e^{iS_{\rm eff}[x^a_\alpha]} = \int \mathcal{D}h^a_{\mu\nu}\mathcal{D}\phi^a e^{i(S_{EH}+S_\phi+S_m)_1 - i(S_{EH}+s_\phi+S_m)_2} \qquad a=1,2$$



Two copies of each field.

The effective action can be decomposed into:

$$S_{\text{eff}}(x^a_\alpha) = \int dt(\mathcal{L}(x^1_\alpha) - \mathcal{L}(x^2_\alpha) + R(x^{1,2}_\alpha))$$

Conservative dynamics

Radiation-reaction interactions

Newton's law becomes:

$$m_{a,b}\ddot{x}_{a,b} = \frac{\partial \mathcal{L}(x_{a,b})}{\partial x_{a,b}} + \frac{1}{2}(\frac{\partial R}{\partial x_1} - \frac{\partial R}{\partial x_2})$$

Dissipation taken into account in a Lagrangian formalism

potential

Radiation-reaction

The corrected potential

$$\lambda \int d^4x\ \phi_0^4 = \int dt\ V_\lambda(x_a, x_b)$$

$$\phi_0 = -\frac{\beta}{4\pi m_{\rm Pl}}\left(\frac{1}{|x-x_a|} + \frac{1}{|x-x_b|}\right)$$

This diverges and needs to be renormalized at the matching scale where the point-like approximation fails. This leads to a classical running of the coupling constant β.

$$V_\lambda(x_a, x_b) = \frac{\lambda\beta^4 G_N^2}{4\pi^2}\left(\frac{6\pi^3 m_a^2 m_b^2}{r} + 4m_a m_b(m_a^2 + m_b^2)\frac{4\pi + \ln(r^2\Lambda^2)}{r}\right)$$

Effects on perihelion of Mercury, Shapiro time delay etc…

Physical log correction

$$\lambda\beta^2 \lesssim (G_N M_\odot^2)^{-1}$$

The long-distance effective action

The effective action is obtained in two steps as before. First integrate the short distance modes:

$$J = J_{\mathrm{matter}} + J_{\mathrm{interaction}}$$

$$J_{\mathrm{interaction}} = \frac{\partial V}{\partial \phi}|_{\phi_0}$$

$$I_\lambda^{1_1 \ldots i_\ell}$$

$$I_\lambda(t) = -\frac{\lambda \beta^3 D \mu M^2}{8\pi m_{\mathrm{Pl}}^3} (\ddot{r^2}),$$
$$I_\lambda^i(t) = -\frac{\lambda \beta^3 m_a m_b}{m_{\mathrm{Pl}}^3} \frac{(2 + \log(27))}{24\pi^2} (m_b - m_a) r^i,$$
$$I_\lambda^{ij}(t) = -\frac{\lambda \beta^3 C \mu M^2}{8\pi m_{\mathrm{Pl}}^3} Q^{ij}.$$

$$C = \left[\frac{26/9 + \log(27)}{5\pi} + \left(\frac{3\log(3)}{\pi} - 2\frac{(2 + \log(27))}{3\pi} \right) \nu \right]$$
$$D = \left[\left(\frac{(26/9 + \log(27))}{6\pi} - \frac{(4 + 9\log(3))}{24\pi} \right) + \frac{1}{3} \left(\frac{3\log(3)}{\pi} - 2\frac{(2 + \log(27))}{3\pi} \right) \nu \right].$$

The effective action in the Keldysh basis reads:

$$\phi_{1,2} = \Phi_{1,2} + \varphi_{1,2}$$

Short distance

Long distance

$$S_{\rm eff}(x^{\pm}_{a,b}) \supset \int d^4x(\delta^{(3)}(x)\sum_n \frac{1}{n!}I_-^{i_1\ldots i_n}\partial_{i_1}\ldots\partial_{i_n}\varphi_+ - 6\lambda\varphi_+^2(\Phi_1^2 - \Phi_2^2)) + \ldots)$$

Dissipated power

Tail effects

$$I_- = I_1 - I_2$$

$$\varphi_+ = \frac{1}{2}(\varphi_1 + \varphi_2), \quad \varphi_- = \varphi_1 - \varphi_2$$

Dissipation:

Direct calculation in the action without taking imaginary parts. Here propagators are retarded in the (+-) basis.

The radiated scalar is given by:

$$\varphi_+ = -\sum_n \frac{1}{n!}\int dt' G_{\rm ret}(\vec{x}, t-t') n_{i_1}\dots n_{i_n}\partial_0^n I_+^{i_1\dots i_n}$$

This can be used to calculate the dissipated action:

$$S_{\rm diss}(x^{\pm}_{a,b}) = \int d^4x(\delta^{(3)}(x)\sum_n \frac{1}{n!}I_-^{i_1\dots i_n}\partial_{i_1}\dots\partial_{i_n}\varphi_+ = -\frac{1}{4\pi}\sum_n \frac{1}{n!(2n+1)!!}\int dt \partial_0^n I_-^{i_1\dots i_n}\partial_0^{n+1}I_+^{i_1\dots i_n}$$

$$P = -\frac{1}{4\pi}\sum_n \frac{1}{n!(2n+1)!!}(\partial_0^{n+1}I^{i_1\dots i_n})^2$$

Coupling between multipole corresponding to the R term

Tail effects:

Memory effects breaking the Markovian property in the interactions: depends on the past!

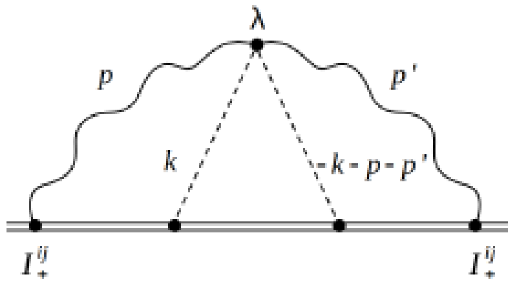


FIG. 4: The diagram associated to the scalar tail effect. There are two radiation fields (wiggly line) coupled to the I_+ source and two conservative fields (dotted lines).

$$S_{\rm tail}(x^{\pm}_{a,b}) \supset -6\lambda \int d^4x \varphi_+^2(\Phi_1^2 - \Phi_2^2))$$

The Feynman integrals give tail terms when logarithms of the energy appear in the integrals… They show up as the integrals are given by Hankel function of zeroth order… The end result is this:

$$\frac{3\lambda\beta^2\mu M}{4(2\pi)^4 m_{\rm Pl}^2}\sum_{m\geq 2}\frac{(-1)^m 2^{n+n'}}{n!n'!(m!)^2 2^{2m}}B(n-m+1/2,n'-m+1/2)\sum_{p'=0}^{m-1-n-n'}(-1)^{p'}C_{2m}^{2p'+1}F(m_a,m_b,p')$$
$$\times\frac{(m-p'-1)!}{(m-p'-n-n'-1)!}\int dt r^{2m-1-n-n'} I_+^{(m),i_1\cdots i_n} n_{i_1}\cdots n_{i_n}\int_{-\infty}^{t}\frac{dt}{t-t'} I_+^{(m),i_1\cdots i_{n'}} n_{i_1}\cdots n_{i_{n'}}.$$

Interaction between multipoles

Same mon-local kernel as in General Relativity between quadrupoles.

Dominated by monopole-monopole interactions.

$F(m_a,m_b,p') = \int_0^1 dy|y-\frac{m_a}{M}|^{2m-2p'-2}(y(1-y))^{p'}$

Force depending in the past

Radiation-reaction

$$\vec{a} = \mathcal{R}\vec{n} \qquad \mathcal{R} = -3\frac{\beta^2\lambda m}{8(2\pi)^4 m_{\rm Pl}^2}B(-1/2,-1/2)\dot{I}\int_{-\infty}^{t}\frac{\dot{I}}{t-t'}$$

This leads to a change of the periastron:

$$\Delta\omega = -\frac{\lambda\beta^2(1-e^2)^{3/2}}{2(2\pi)^3}B(-1/2,-1/2)C(e)\Delta\omega_{\rm GR}$$

Fourier modes of monopole

$$J_{n,n'} = e(J_{(n-n')}\left((n-n')e\right) - J_{(n+n')}\left((n+n')e\right))$$
$$-\frac{1}{2}(J_{(n-n')+1}\left((n-n')e\right) + J_{(n-n')-1}\left((n-n')e\right) - J_{(n+n')+1}\left((n+n')e\right) - J_{(n+n')-1}\left((n+n')e\right)).$$

$$C(e) = \frac{1}{e}\sum_{n>0,n'>0} nn'\ln(2\pi|n'|)I_n I_{n'} J_{n,n'}$$

$$\frac{\Delta\omega}{\Delta\omega_{\rm GR}} \lesssim \beta^2 \left(\frac{G_N m}{a}\right)^2$$

Observable effect if large coupling to matter and large Newtonian potential…..

Summary

- Scalar effects can test the existence of interactions to matter and self-interactions:

Kicks and memory for hyperbolic orbits (white dwarfs?)

- Heavily dependent on the coupling to matter β

β =0 for black holes… (no hair) unless time-dependence effects

β of order unity for scalarised model?

- Heavily dependent on self-coupling λ

$$\lambda \lesssim \frac{1}{\beta^2 G_N M_\odot^2}$$

Bullet cluster (dark matter):

$$\lambda \lesssim 10^{-12}(\frac{m}{\mathrm{eV}})^2, \quad m \lesssim 10^{-20}\mathrm{eV}$$

Dark energy: $\lambda \sim \frac{V_0}{m_{\mathrm{Pl}}^4} \sim 10^{-120}$

Typically one expects that the conformal and disformal interactions are small corrections to the Newtonian (or GR) case. Our treatment is only for small Newtonian potentials, i.e. way outside the Schwarzschild radius, and moreover we must require:

$$\phi \simeq -\frac{\beta m}{4\pi m_{\rm Pl} r}$$

$$\phi \leq m_{\rm Pl} \implies r \geq 2\beta G_N m$$

Always satisfied (Cassini bound)

The disformal interaction will not exceed the conformal interaction provided:

$$r \geq r_\star = \sqrt{\frac{\beta m}{4\pi m_{\rm Pl^2} \Lambda}}$$

If we want to use a perturbative treatment of the scalar interaction down to a few hundred km's from objects of around 1 solar mass like neutron stars in orbits and a Yukawa coupling at the Cassini bound:

$$r_\star \leq 10^2 r_{s\odot} \implies \Lambda \geq 10^{-17}\ {\rm eV}$$

Could be much lower for small Yukawa couplings.